BLOOD FLOW, SLIP, AND VISCOMETRY

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ABSTRACT The viscosity of blood, measured by the usual viscometers in which slip is not considered, is found to be flow dependent, varying markedly with shear rate, pressure gradient, and vessel diamcter in the lower ranges of these factors. The study postulates, on grounds thought reasonable, that slip may be present in blood flow, as a function of the nature of the wall surfaces, shear stress at the wall, and relative cell volume (RCV) adjacent to the wall. It presumes that blood possesses a specific, flow-independent viscosity, and determines theoretically the viscosity indications of viscometers if blood slipped in the instruments. The study shows that if the slip function is of a certain plausible form, these viscosity indications would exhibit a flow dependence of much the same pattern as the actual indications supplied by the usual viscometers. The slip postulate permits, therefore, an interpretation of the "anomalous" flow behavior of blood, dispensing with the prevailing assumption of an ad hoc variability of its viscosity with flow factors. To the extent that viscometric data for blood may be representative of other non-newtonian fluids, the slip postulate may be applicable to these fluids.

INTRODUCTION

Slip is defined as a finite velocity of a fluid at a boundary or in its immediate vicinity. A widely used hypothesis in fluid mechanics is that fluids flow without slip; that is, they adhere to the boundary and their velocity profile is continuous in the adjacent region. The "no slip" hypothesis has worked well in the study of newtonian fluids. In the case of non-newtonian fluids, especially suspensions, it has been found questionable by some researchers.

This has come about as a result of comparatively recent studies, both experimental and theoretical, of the flow of fluids, mainly blood.[1] Some of these studies state, others suggest or imply, that slip cannot be ruled out as a significant element in the understanding of certain flow peculiarities (2–7).[2] Isenberg (9) posits slip "for all practical purposes" in his study of blood flow in capillary tubes. Bennett (10) reports that microscopic examination of blood flowing past a glass wall shows slipping ("skidding") of red cells in contact with the wall.[3] The same phenomenon has been observed in vivo by Bloch (11). Theoretical studies of tubular flow of blood

[1] The no slip hypothesis of fluid mechanics has been questioned earlier in connection with the flow of gases at very low densities (1).

[2] For a more detailed review of opinions regarding slip, see reference 13. For an historical summary of views on slip, see reference 8.

[3] The contact is an "optical" one (resolution $>$ 0.04 microns).

in which slip was allowed as a function of shear stress have also accounted for several flow characteristics observed experimentally (12, 13).

It would appear that the presence of slip in viscometers must be accepted as a possibility. To study the effect of possible undetected slip in these instruments upon viscosity measurements, a brief description of viscometers and their operation will be helpful.

Two principal types of viscometers are in general use, representative of most other varieties. They are essentially the following:

(*a*) Coaxial rotating cylinders, in which the fluid is sheared in a thin gap between two cylinders, the inner one being generally the rotor. Shear stress (τ) and shear rate ($\gamma^{\bullet}$) are proportional to the measured torque and the observed angular velocity, respectively. Viscosity is calculated as the ratio of shear stress to shear rate, that is as $\tau/\gamma^{\bullet}$. A typical family of viscosity vs. shear rate curves for blood of different hematocrits is shown in Fig. 1 (adapted from Fig. 1 of reference 14) and in Figs. 2 and 3 (adapted from Fig. 3 of reference 15).

(*b*) Capillary tubes, in which the fluid is made to flow under conditions of laminar flow. The pressure gradient ($\Delta p/l$) and the flow rate ($Q^{\bullet}$) are measured, and viscosity is computed from the Poiseuille flow relations as $[(\pi r^4/8)(\Delta p/l)]/Q^{\bullet}$ (r being the tube radius). Viscosity may then be plotted as a function of pressure gradient. A more usual equivalent plot is that of flow rate vs. pressure gradient. A typical family of such curves is shown in Fig. 4 (adapted from Fig. 1 of reference 16. See also Fig. 1 of reference 2).

In capillary flow, the viscosity of blood is found to vary with the tube radius, decreasing with it as shown by the curves of Fig. 5 (adapted from Fig. 2 of reference 17. See also Fig. 1 of reference 18). This is referred to as the Fåhraeus-Lindqvist effect in very thin tubes.

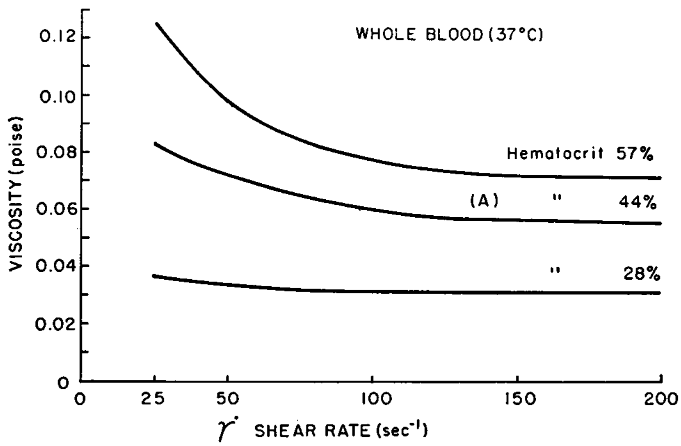

FIGURE 1 Blood viscosity as a function of shear rate; experimental results. (Adapted from Fig. 1 of reference 14.)

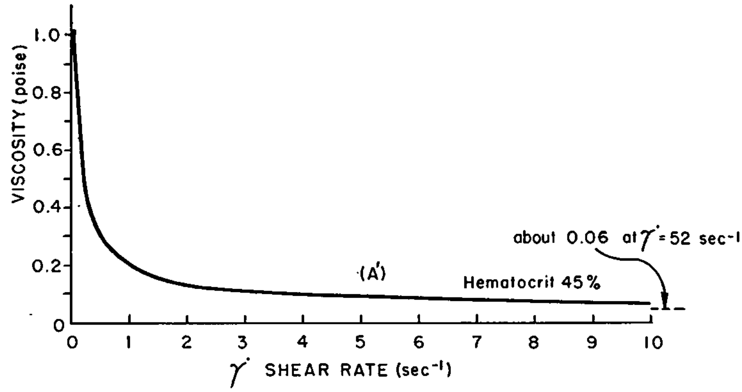

FIGURE 2 Blood viscosity as a function of shear rate; experimental results. (Adapted from Fig. 3 of reference 15.)

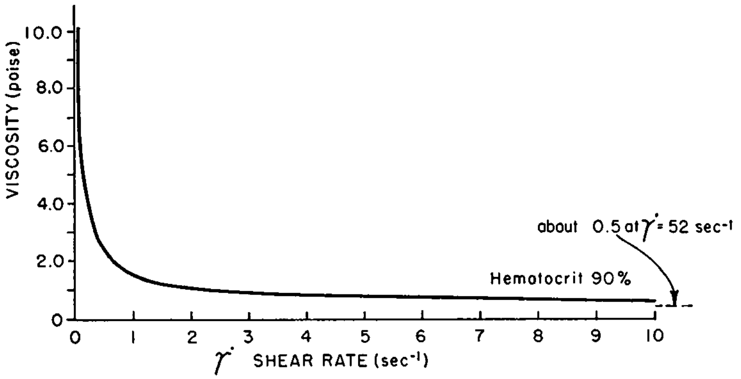

FIGURE 3 Blood viscosity as a function of shear rate; experimental results. (Adapted from Fig. 3 of reference 15.)

The viscosity measurements made in these instruments concern viscosity "in the large" and are referred to as "apparent" or "indicated" viscosity.

As illustrated by Figs. 1–5, experimental findings about blood flow are characterized by a marked dependence of apparent viscosity upon flow factors, such as shear rate, pressure gradient, and vessel diameter, in the region of small values of these factors. Although there is, on the whole, poor quantitative agreement among existing experimental results,[4] there is concordance as to general character and pattern. These figures are therefore typical for available experimental findings about

[4] Charm and Kurland (19) report that blood viscosity measured by a Couette viscometer was lower than measurements by either cone-and-plate or capillary viscometer by 10–20%; for blood of 20% hematocrit, also for plasma, there seemed to be little or no discrepancy. McDonald (20) mentions still larger discrepancies in measurements by various observers.

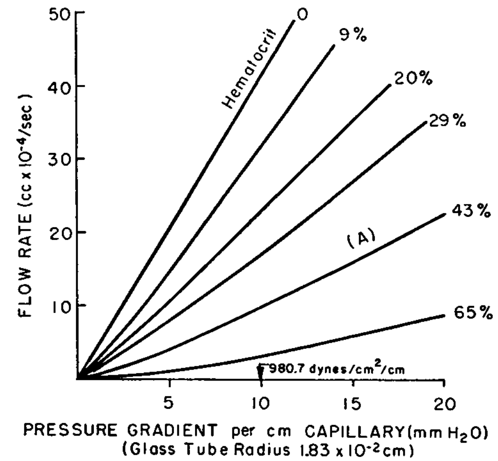

FIGURE 4

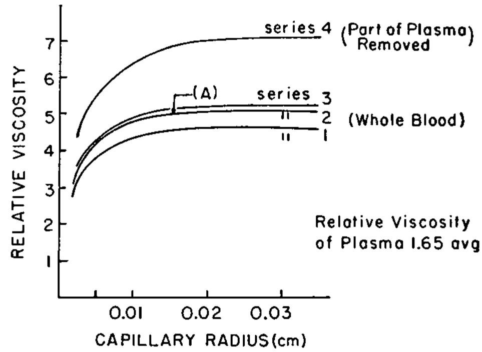

FIGURE 5

FIGURE 4 Flow rate of blood as a function of pressure gradient; experimental results. (Adapted from Fig. 1 of reference 16.)

FIGURE 5 Relative viscosity of blood (with respect to water) as a function of capillary tube radius; experimental results. (Adapted from Fig. 2 of reference 17.)

blood rheology. They illustrate the anomalous, or non-newtonian, flow behavior of blood, and of certain other non-newtonian fluids (5).[5]

Various explanations have been advanced for the anomalous flow of blood. It has been related to the deformability and aggregation of red cells (21–24). Other interpretations are based on the orientation of red cells along the lines of flow due to their discoid shape, also to their axial drift in tubular flow (2, 16, 25, 26. For a review of these and other interpretations, see reference 27). It may be noted that non-newtonian flow properties are not restricted to fluids with a deformable, clumping or shaped phase (5).

When faced with anomalous flow behavior one may in general choose between two types of approach: (*a*) the use of a nonlinear shear stress-shear rate relationship, [for instance, the "power law" $\tau = k(\dot{\gamma})^n$, (5)], of some form varying with the flow condition, together with the no slip hypothesis; (*b*) the use of a constant (flow-independent) viscosity, in the manner of other physical constants, with slip at the boundaries. (To some extent, these choices are not mutually exclusive.) The first choice is made in almost all existing theoretical investigations of viscometric flows. This study makes the second choice for reasons to be presented shortly.

GENERAL DESCRIPTION

The study postulates that slip may be present in blood flow, its magnitude depending on the nature of the wall surfaces, shear stress at the wall, and relative cell volume

[5] It is not clear whether the Fåhraeus-Lindqvist effect (Fig. 5) is found in non-newtonian fluids in general, or is specific to suspensions, or to blood alone.

(RCV) in the region immediately adjacent to the wall. It presumes that blood possesses a specific, flow-independent viscosity, (variable of course with hematocrit, temperature, etc.), and proposes to interpret its anomalous flow behavior on the basis of slip.

Although plasma itself is known to be newtonian and nonslipping, in flowing blood the skidding red cells in contact with the wall mentioned earlier (10, 11) may be expected to entrain by their momentum some amount of plasma in the periphery. The fluid would thus slip to some extent, depending on the RCV at the periphery, flow velocity (equivalently, shear stress), and the nature of the wall surfaces. A slip postulate as formulated above therefore suggests itself.

Moreover, considering tubular flow, the axial drift of red cells referred to earlier produces a very thin peripheral zone, or sheath, characterized by a low RCV in comparison with the core, and in which RCV for a given hematocrit decreases with increasing shear (2, 25, 26). The consequent decrease in the number of skidding cells at the wall should, from the point of view of this study, decrease slippage in the sheath and make the flow more and more newtonian as shear increases. This is observed experimentally (see Figs. 1–4). Conversely, for a given shear, the sheath is known to decrease in thickness with increasing hematocrit (25), with a consequent increase of RCV in the sheath and the number of skidding cells at the wall. This should, again from the viewpoint of this study, increase slippage in the sheath and make the flow increasingly anomalous as hematocrit increases. This too is observed experimentally (see Figs. 1–4).

From the preceding general remarks, the slip postulate appears plausible. This is further suggested by the views about slip reported in the Introduction; also supported, as it is felt, by the general agreement of the results of the study with existing experimental data on viscometric flow. The work proceeds as follows.

By definition, viscosity μ is the ratio (shear stress):(shear rate, slip excluded). If blood slips in the viscometer, and the fact is not taken into account, the instrument will indicate a viscosity μ_i calculated as the ratio (shear stress):(shear rate, inclusive of slip). These two values are compared. If the presumed slip is a certain reasonable function of the shear stress at the wall, and if for μ is used the experimental value of viscosity at very low shear rate—therefore, little affected by viscometer slip, if present—then the indicated viscosity μ_i, determined theoretically, is found to vary with shear rate, pressure gradient, and vessel diameter in the manner illustrated by the experimental findings of Figs. 1–5.

The anomalous flow behavior of blood may therefore be interpreted by means of the slip postulate, rather than by attributing to its viscosity, as is done currently, an ad hoc variability with flow factors.

Due to the lack of empirical data concerning slip, the form of its functional dependence upon shear stress at the wall can only be conjectured as a first approximation on the basis of considerations of simplicity and general suitability.

The two simplest forms for a slip function are the linear $s = b\tau$ and the parabolic quadratic $s = a\tau^2 + b\tau$, where s is the slip, τ the shear stress at the wall, and a, b two positive factors depending on the nature of the wall surfaces and the RCV in the peripheral zone. However, these forms may apply only for very low shear stresses, and this would rule out attempts to discuss flow properties where relatively high shear stresses are involved (e.g., Fåhraeus-Lindqvist effect). Moreover, they are found unsuitable for not meeting general requirements, as will be seen in the subsequent discussion.

Next in order of complexity, we consider the hyperbolic quadratic $s = a\tau^2/(1 + b\tau)$ a two-parameter form of essentially simple character, even if not of the more familiar polynomial type. As can be seen easily, it is approximated by the parabola $s = a\tau^2$ for small shear stresses τ, and by the straight line $s = a\tau/b$, when they are large. Slip in this form would start from zero and build up slowly as shear increases—the representative curve being tangent to the τ axis at the origin—then accelerate to become asymptotically a near-linear function of shear stress in the high ranges (that is, slip increases indefinitely with shear stress, but the ratio [slip] : [shear stress] remains finite). The form seems in general to satisfy an a priori mental image of how slip, if it existed, ought to develop with increasing shear stress, and proves practicable. It is adopted here for slip function as a first approximation. Fig. 6 illustrates a function of this type.

Concerning the selection of numerical values for the factors a and b, all that is possible in the circumstances is to give them a number of "likely" magnitudes by assigning to the asymptotic slope a/b (Fig. 6) of the slip function curve a succession of simple values, thereby varying the "flatness" of the function, hence the amount of slippage, over a fairly wide range.

Each pair of values a, b defines a slip function of the form specified; therefore, a relationship between indicated viscosity and the flow factors—shear rate, pressure gradient, tube radius, as the case may be. A number of such relations are developed and plotted in Figs. 7–9 for comparison with existing experimental data for blood, Figs. 1–5.

The two types of viscometers described earlier are discussed in the following on

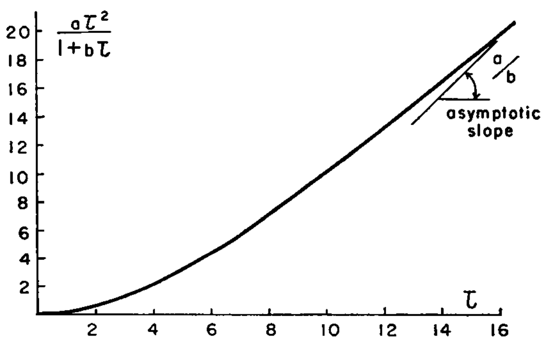

FIGURE 6 Typical function $a\tau^2/(1 + b\tau)$, ($a = 0.2$; $b = 0.1$; undefined units.)

the basis of the slip postulate stated above. The discussion makes use of the same form of slip function for both types, thus unifying the interpretation of capillary tube data and Couette data.

Experimental results for blood are available in far larger volume and detail than for other non-newtonian fluids. To the extent that the blood data may be representative of these fluids (a likelihood, at least for suspensions), the present study may be applicable to them.

FLOW IN VISCOMETERS

Coaxial Rotating Cylinders Viscometer

The fluid, possessing by hypothesis a specific, flow-independent viscosity μ, is flowing in the viscometer and assumed to be slipping. Slip, denoted by σ, is to be thought of here as the velocity of the fluid at the cylinder surfaces divided by the thickness of the gap between the cylinders. It is recalled that τ and $\gamma^{\bullet}$ stand for the shear stress at the boundary and the shear rate, respectively.

As noted earlier, in viscometric measurements the apparent viscosity of the fluid is calculated by the ratio $\mu_i = \tau/\gamma^{\bullet}$. This assumes that the fluid is sheared at the rate $\gamma^{\bullet}$. Actually, if slip is present, as assumed here, the true shear rate is $\gamma^{\bullet}$ less two slips σ (one at each fluid-cylinder interface) and the true viscosity of the fluid is defined by $\mu = \tau/(\gamma^{\bullet} - 2\sigma)$. Eliminating τ between these two relations, there follows

$$\frac{\mu_i}{\mu} = 1 - \frac{2\sigma}{\gamma^{\bullet}}. \tag{1}$$

For slip function the two simple forms $\sigma = \beta\tau$ and $\sigma = \alpha\tau^2 + \beta\tau$, mentioned earlier, are considered, α and β being two factors of the same character as factors a and b previously defined. The quadratic is tried first. Its use in equation 1, after τ is replaced by $\mu_i\gamma^{\bullet}$, gives the relation

$$2\,\alpha\mu\gamma^{\bullet}\mu_i^2 + (1 + 2\beta\mu)\mu_i - \mu = 0, \tag{2}$$

which furnishes μ_i as a function of $\gamma^{\bullet}$. It shows that as $\gamma^{\bullet}$ decreases toward zero, μ_i tends toward $\mu/(1 + 2\beta\mu)$, instead of approaching μ, as it obviously should. The relation also shows that μ_i decreases toward zero as $\gamma^{\bullet}$ increases indefinitely, instead of leveling off quickly toward some finite value, as it is known to do experimentally. The form $\sigma = \alpha\tau^2 + \beta\tau$ is therefore unsuitable as a slip function. The linear form $\alpha = \beta\tau$ is equally inappropriate, as could be shown in the same manner.

The following form, discussed earlier, is used instead for slip function:

$$\sigma = \frac{\alpha\tau^2}{1 + \beta\tau}. \tag{3}$$

The expression is introduced into equation 1, after τ is replaced by $\mu_i\gamma^\bullet$. The resulting relation

$$(2\,\alpha\mu + \beta)\gamma^\bullet\mu_i^2 + (1 - \beta\mu\gamma^\bullet)\mu_i - \mu = 0, \qquad (4)$$

defines μ_i as a function of $\gamma^\bullet$, in the presence of slip specified by equation 3.

Equation 4 shows that μ_i tends toward μ as $\gamma^\bullet$ decreases toward zero, and toward a finite value $\beta\mu/(2\alpha\mu + \beta)$ as $\gamma^\bullet$ increases indefinitely, all of which agrees with experiment.

For the numerical processing of equation 4, the factors α, β and viscosity μ are specified as follows:

For α and β, a series of simple values are assigned to the asymptotic slope α/β of the slip function, as indicated earlier, namely, 2, 10, 50; α, β are then given a succession of magnitudes bearing these ratios to each other. Thus, the following sets are used successively in equation 4: $\alpha = 2 \times 10^{-1}, \beta = 1 \times 10^{-1}$; $\alpha = 1.0, \beta = 1 \times 10^{-1}$; $\alpha = 5.0, \beta = 1 \times 10^{-1}$; $\alpha = 1.0, \beta = 5 \times 10^{-1}$; $\alpha = 1.0, \beta = 2 \times 10^{-2}$. It will be seen that the resulting family of curves covers a wide band of the plot space.

As regards specific viscosity μ, it is logical from the point of view of this study to ascribe to it that experimental value of viscosity which would have been least affected by a presumed slip in the viscometer, the value therefore which corresponds to the lowest shear rate used in available experiments. In Fig. 4 of reference 15, the lower end of the shear rate range is 0.01 sec^{-1} (for blood of 51.7 % hematocrit), in the neighborhood of which a viscosity plateau is indicated.[6] In Fig. 3 of the same work, extrapolation toward zero shear rate of the viscosity curve for 45 % hematocrit suggests, assuming a similar plateau, a viscosity of about 100 centipoises—quite a high figure, incidentally, in comparison with the usual values for blood of this hematocrit. For the numerical processing of equation 4, the fluid is taken to be blood of 45 % hematocrit—a usual value—of specific viscosity μ equal to 1 poise (cgs unit), in accordance with the above discussion.

With the values α, β, and μ specified above, equation 4 is computed and plotted in the form of a family of curves for μ_i as a function of $\gamma^\bullet$ (Fig. 7). The curves illustrate the influence of a presumed slip upon viscometric indications, and are seen to be of the same general pattern as the empirical curves of Figs. 1–3. For a closer comparison, curve A, $\gamma^\bullet$ range 25–200, and curve A′, $\gamma^\bullet$ range 0–52, from Figs. 1 and 2 (44 and 45 % hematocrit), are shown dotted in Fig. 7; they are seen lying close to one of the theoretical curves of the family over the entire $\gamma^\bullet$ range. Better concordance could be achieved by giving the α, β factors proper magnitudes determined by computer. However, in order to avoid possible criticism for achieving agreement by curve-fitting processes (a practice considered suspect by some), preference was

[6] Possible interpretation of the plateau: no slip up to a shear threshold in the neighborhood of 0.01 sec^{-1}. In this connection, it may be noted that no yield stress was observed in blood at very low shear rate by reference 15.

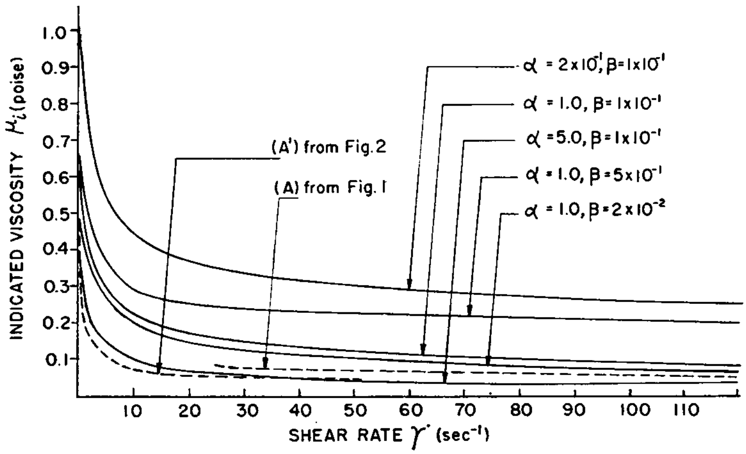

FIGURE 7 Indicated blood viscosity as a function of shear rate; theoretical results.

given to holding these factors as a sequence of round numbers and demonstrating that the resulting curves are of the same character and pattern as existing data, and the latter are bracketed by the curves. (This remark applies also to Figs. 8 and 9 to follow.) All computations were made with the standard 12 inch slide rule, the round-off errors being as a rule those normally incurred in the use of these instruments.

In the absence of slip ($\alpha = 0$), equation 4 gives $\mu_i = \mu$, a constant, for all values of the shear rate $\gamma^{\bullet}$, and the flow is newtonian.

Capillary Tube Viscometer

The fluid, assumed to possess a specific, flow-independent viscosity μ, is again flowing in the viscometer, and slip is supposed present. Symbol s denotes the slip, and $Q^{\bullet}$, $\Delta p/l$, τ, r are respectively the flow rate, pressure gradient, shear stress at the wall and tube radius, as defined earlier. The mean velocity of flow without slip, under the same conditions of flow otherwise, is denoted by $\bar{u}$.

The following relations are established in the same manner as the usual ones for a Poiseuille flow, (to which they reduce for $s = 0$):

$$Q^{\bullet} = \left(\frac{1}{8\mu}\frac{\Delta p}{l}r^2 + s\right)\pi r^2,$$

$$\bar{u} = \frac{1}{8\mu}\frac{\Delta p}{l}r^2,$$

$$\tau = \frac{r}{2}\frac{\Delta p}{l}. \qquad (5)[7]$$

[7] It may be noted that shear stress at the wall is independent of slip.

These relations give

$$Q^{\bullet} = \left(1 + \frac{s}{\bar{u}}\right)\frac{\Delta p}{l} \times \frac{\pi r^4}{8\mu}. \tag{6}$$

In tubular flow, the apparent or indicated viscosity μ'_i (primed, to distinguish it from μ_i, that of the rotating viscometer discussed before) is defined as proportional to $(\Delta p/l)/Q^{\bullet}$ (25), with $\pi r^4/8$ as proportionality factor (this is equivalent to the primary definition of viscosity as the ratio of shear stress to shear rate). Introducing this into equation 6, there follows

$$\frac{\mu_i'}{\mu} = \frac{1}{1 + \frac{s}{\bar{u}}}. \tag{7}$$

The slip function, of the same form as relation 3 used earlier, is the expression

$$s = \frac{a\tau^2}{1 + b\tau}, \tag{8}$$

in which factors a and b are positive numbers depending on the nature of the wall surfaces and the RCV in the peripheral zone of the fluid. Using equation 8 in equations 6 and 7, and taking account of equation 5, there follows

$$Q^{\bullet} = \left[\frac{1}{\mu} + \frac{2a\frac{\Delta p}{l}}{1 + \frac{1}{2}br\frac{\Delta p}{l}}\right] \times \frac{\Delta p}{l} \times \frac{\pi r^4}{8}, \tag{9}$$

and

$$\frac{1}{\mu'_i} = \frac{1}{\mu} + \frac{2a\frac{\Delta p}{l}}{1 + \frac{1}{2}br\frac{\Delta p}{l}}. \tag{10}$$

If slip is absent ($a = 0$), equation 10 gives $\mu'_i = \mu$, a constant, for all values of the pressure gradient, and the flow is newtonian.

As in the case of equation 4, equations 9 and 10 are processed numerically for comparison with existing experimental data. Again, the fluid is assumed to be blood of 45 % hematocrit, and specific viscosity μ is taken as 1 poise.

Referring to a preceding discussion of the axial drift of red cells and the resulting peripheral sheath in which RCV is known to decrease with increasing shear, it is evident that factors a and b, depending by definition on RCV in the sheath, itself variable with shear, must be functions of shear stress. How they vary with it will not be presumed here, beyond specifying that a is to decrease as shear stress, or pressure gradient, increases. To proceed with the numerical work in a fairly rough, but as simple a manner as possible, two sets of a, b values for the factors will be used: one set when the pressure gradient is below some reasonable level, the other set when it is above that level.

Once more with the thought of varying the flatness of the slip function over a fairly wide range, and for the purpose of bringing the computed results into the range of existing experimental data, the two sets of a, b values are derived from the values used earlier for the factors α, β as follows: set a', b' defined by $a' = 4 \times 10^{-8}\,\alpha$, $b' = \beta$ for use when $\Delta p/p$ is less than 2000 dynes/cm^2 per cm; set a'', b'' defined by $a'' = 10^{-4}\,\alpha$, $b'' = \beta$ for use when $\Delta p/p$ exceeds the above value. These sets are listed in Figs. 8 and 9.

In the computation of equation 9, the range of pressure rate is taken as 0–20 mm H_2O/cm (equal to 2000 dynes/cm^2 per cm, about) and the tube radius as 1.83×10^{-2} cm, these figures being the magnitudes used in Fig. 4 with which the numerical results will be compared. Using then the a', b' values for a, b in equation 9, a number of curves are plotted and shown in Fig. 8, illustrating the influence of a presumed slip upon rate of flow. They exhibit the characteristic form of variation of flow rate with pressure gradient, pictured by the experimental diagrams of Fig. 4. For closer com-

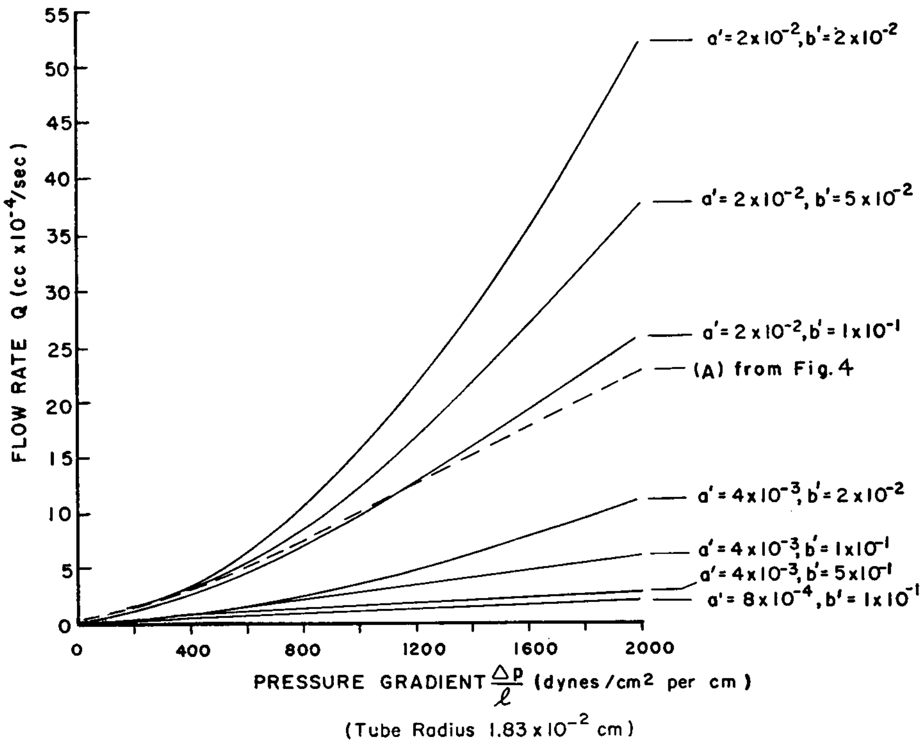

FIGURE 8 Flow rate of blood as a function of pressure gradient; theoretical results.

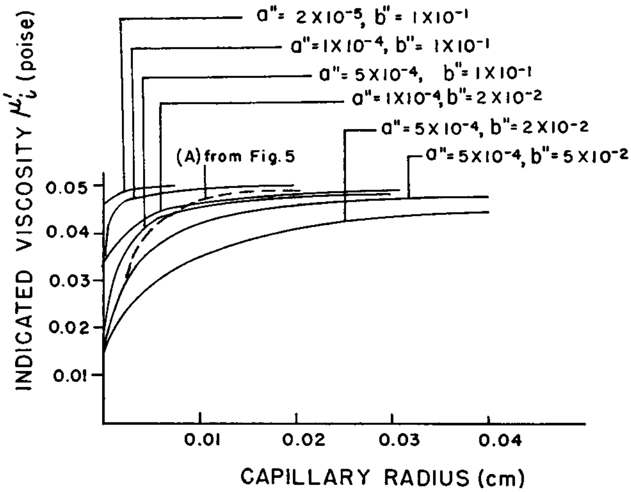

FIGURE 9 Indicated viscosity of blood as a function of capillary tube radius; theoretical results.

parison, curve A of that figure (43% hematocrit) is shown dotted in Fig. 8, and it is seen to lie close to one of the theoretical curves. As noted earlier in connection with Fig. 7, a closer fit could be achieved easily by using for a' and b' magnitudes other than round numbers.

Equation 10 is computed and plotted as a family of curves in Fig. 9. The tube radius is now the variable, the pressure gradient being a constant (taken here as 5×10^4 dynes/cm^2 per cm, average of the high experimental values used in reference 17,[8] the source of Fig. 5 with which the computed results will be compared). For the viscosity term μ in equation 10 is now used the value 0.05 poise instead of the earlier value $\mu = 1$. The reason for this is that slip in tubes of varying radius in high shear regions may be considered as having occurred in two stages: first, as a result of pressure rate varying from zero to high levels—the tube radius remaining constant—during which stage apparent viscosity drops from 1 to 0.05 poise (approximately), as shown in Fig. 7; second, as a result of the tube radius being made to vary—pressure rate now remaining constant. The base value for viscosity during this second stage must therefore be taken as 0.05 poise. For a, b in equation 10 the set a'', b'' is used, as indicated earlier. The curves of Fig. 9 illustrate the effect of a slip upon indicated viscosity in very thin tubes. They exhibit the characteristic Fåhraeus-Lindqvist effect observed in such tubes, seen in Fig. 5. To allow a close comparison, curve A of the latter figure is indicated dotted in Fig. 9, and it is seen to be bracketed by some of the theoretical curves. To transfer the viscosity curve A (relative to water) to Fig. 9, the viscosity of water was taken as 0.01 poise.

[8] These values vary from 1×10^4 to 1×10^5 dynes/cm^2 per cm, about. The test results are not sensitive to pressure gradient variations in these high ranges; this is also true for equation 10, as can be seen easily.

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